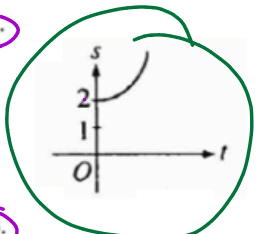
Period 1, April 7, 2025

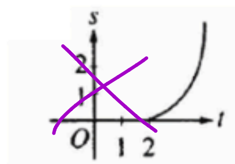
$V(0)=0$ Time $S(0)=2$ $(0,2)$

1. **1998 #90 (BC) - Calc OK:** A particle starts from rest at the point (2, 0) and moves along the x-axis with a constant positive acceleration for time $t \geq 0$. Which of the following could be the graph of the distance $s(t)$ of the particle from the origin as a function of time t?

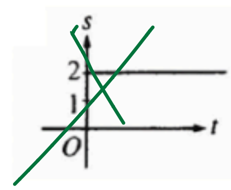
a.



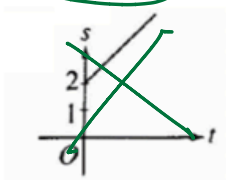
c.



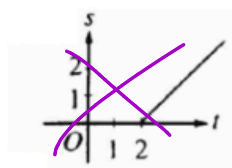
e.



b.



d.



$S(T)$ = distance
$S'(T) = V(T)$ = velocity
$S''(T) = a(T)$ = acceleration

$\underbrace{S''(T) = \text{positive}}$

means $S(T)$ is concave up

10. **1998 #24 (AB but suitable for BC) - No Calc:** The maximum acceleration attained on the interval $0 \leq t \leq 3$ by the particle whose velocity is given by $v(t) = t^3 - 3t^2 + 12t + 4$ is

a. ~~9~~ b. 12 c. 14 d. 21 e. 40

$V'(T) = a(T) = 3T^2 - 6T + 12$

$a'(T) = 6T - 6$

$T = 1$

$a(1) = 3(1)^2 - 6(1) + 12 = 9$

$a(0) = 3(0)^2 - 6(0) + 12 = 12$

$a(3) = 3(3)^2 - 6(3) + 12$

$27 - 18 + 12 = 21$

Test Endpoints

14. **1997 #13 (BC) - No Calc:** A particle moves along the x-axis so that its acceleration at any time t is $a(t) = 2t - 7$. If the initial velocity of the particle is 6, at what time t during the interval $0 \leq t \leq 4$ is the particle farthest to the right?

$T=0$
$V(0)=6$

a. 0 b. 1 c. 2 d. 3 e. 4

$V(T) = \int a(T)\,dT$

$\int (2T-7)\,dT = T^2 - 7T + C = V(T)$

$0^2 - 7(0) + C = 6$

$C = 6$

$V(T) = T^2 - 7T + 6$

$= (T-1)(T-6)$

cut off

Right

$+ + + + \; - - - - - - - - - - - - \; + + +$

0 1 2 3 4 5 6 7

Test (0, 3, 7)

2017

5. Two particles move along the x-axis. For $0 \le t \le 8$, the position of particle P at time t is given by $x_P(t) = \ln\left(t^2 - 2t + 10\right)$, while the velocity of particle Q at time t is given by $v_Q(t) = t^2 - 8t + 15$. Particle Q is at position $x = 5$ at time $t = 0$. Part d

(a) For $0 \le t \le 8$, when is particle P moving to the left? $(0,1)$

(b) For $0 \le t \le 8$, find all times t during which the two particles travel in the same direction. $(1,3) \cup (5,8)$

(c) Find the acceleration of particle Q at time $t = 2$. Is the speed of particle Q increasing, decreasing, or neither at time $t = 2$? Explain your reasoning.

(d) Find the position of particle Q the first time it changes direction.

(b) $V_Q(T) = (T-3)(T-5)$

Test $(0, 4, 6)$

$+ + + + + +$ $- - -$ $+ + +$

$0 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6$

change direction

Test 0 and 3

Left $- - -$ Moving Right $+ + + + + + +$

$0 \quad 1 \quad 2 \quad 3 \quad 5 \quad 6$

(a) $V_P(T) = \dfrac{1}{T^2-2T+10} \cdot (2T-2) = \dfrac{2T-2}{T^2-2T+10} = \dfrac{2(T-1)}{T^2-2T+10}$

$\sqrt{(-2)^2 - 4(1)(10)}$

$\sqrt{-36}$

can't be factored

always positive

(c) $V_Q(T) = T^2 - 8T + 15$

$a_Q(T) = 2T - 8$

$a_Q(2) = 2(2) - 8 = -4$

$V_Q(2) = 4 - 16 + 15 = +3$

different signs

Slowing down

(d) $V_Q(T) = T^2 - 8T + 15$

$T = 0 \quad S(0) = 5$

$S_Q(T) = \int (T^2 - 8T + 15)\,dT = \frac{1}{3}T^3 - 4T^2 + 15T + C = S(T)$

$5 = \frac{1}{3}(0)^3 - 4(0)^2 + 15(0) + C$

$C = 5$

$S(T) = \frac{1}{3}T^3 - 4T^2 + 15T + 5$

$S(3) = \frac{1}{3}(3)^3 - 4(3)^2 + 15(3) + 5 = 9 - 36 + 45 + 5 = 23$

The data in the table below give selected values for the velocity, in meters/minute, of a particle moving along the x-axis. The velocity v is a differentiable function of time t.

↳ continious as well

Time t (min)	0	2	5	6	8	12
Velocity $v(t)$ (meters/min)	-3	2	3	5	7	5

1. At $t = 0$, is the particle moving to the right or to the left? Explain your answer.

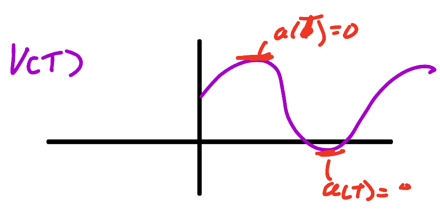
2. Is there a time during the time interval $0 \le t \le 12$ minutes when the particle is at rest? Explain your answer.

3. Use data from the table to find an approximation for $v'(10)$ and explain the meaning of $v'(10)$ in terms of the motion of the particle. Show the computations that lead to your answer and indicate units of measure.

4. Let $a(t)$ denote the acceleration of the particle at time t. Is there guaranteed to be a time $t = c$ in the interval $0 \le t \le 12$ such that $a(c) = 0$? Justify your answer.

V(T) is a smooth curve

So V(T) slope changes sign then a(T) = 0



(6, 5), (12, 5)

$$\frac{5-5}{12-6} = \frac{0}{6} = 0$$

Rolles Theorem

College Board (calc)

Mary and Chance walk in the same direction along a straight path. For $0 \le t \le 20$, Mary's velocity at time t is given by $M(t) = \frac{6010}{t^2 - 3t + 50.5}$ and Chance's velocity at time t is given by $C(t) = 8.5t^3 e^{-0.45t}$. Both $M(t)$ and $C(t)$ are positive for $0 \le t \le 20$ and are measured in meters per minute, and t is measured in minutes. Mary is 12 meters ahead of Chance at time $t = 0$, and Mary remains ahead of Chance for $0 < t \le 20$.

(a) Find the value of $\frac{1}{10}\int_5^{15} M(t)\,dt$ Using correct units, interpret the meaning of $\frac{1}{10}\int_5^{15} M(t)\,dt$ in the context of the problem.

(b) At time $t = 10$, is Mary speeding up or slowing down? Give a reason for your answer.

(c) Is the distance between Mary and Chance at time $t = 18$ increasing or decreasing? Give a reason for your answer.

is Always ahead

who Has The GreaTesT $V(T)$

(d) What is the maximum distance between Mary and Chance over the time interval $0 \le t \le 20$? Justify your answer.

$$\int_5^{15} \left(\frac{6010}{x^2 - 3x + 50.5} \right) dx = 544.091674739$$

(a) $\frac{1}{10}\int_5^{15} M(T)\,dT = \frac{544.092}{10}$

54.4092 m/min

Average velociTy

(b) $M(T) = \frac{6010}{T^2 - 3T + 50.5}$

$M'(T) = 6010(2T-3)(T^2-3T+50.5)^{-2}$

$M'(T) = -1 \cdot 6010(2(10)-3)(10^2 - 3(10) + 50.5)$

$M(10) =$ PosiTive

$M'(10) =$ NegaTive

different signs

"slowing down"

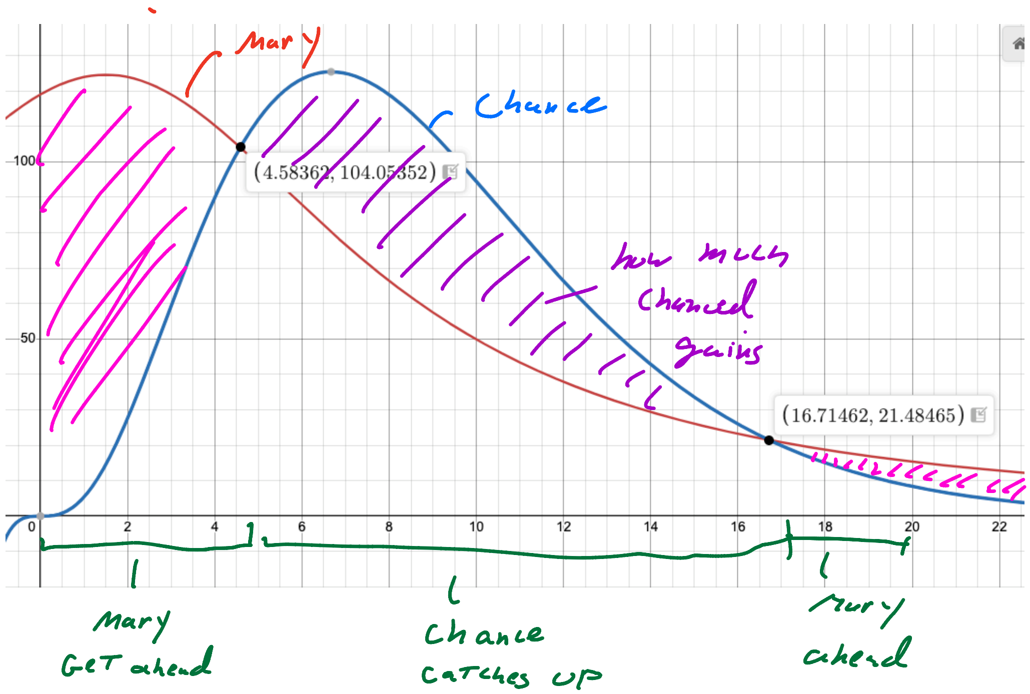
(c) $M(18) = \frac{6010}{18^2 - 3(18) + 50.5} = 18.752$

$C(18) = 8.5(18)^3 e^{-.45(18)} = 15.047$

Mary is in FronT with a GreaTer velociTy

Mary is geTTing FurTher away

(d) when is Mary's Largest Lead



$$\int_{0}^{4.583}\left[\left(\frac{6010}{x^2-3x+50.5}\right)-8.5x^3e^{-.45x}\right]dx = 353.527463175$$

$$\int_{4.583}^{16.7}\left[\left(\frac{6010}{x^2-3x+50.5}\right)-8.5x^3e^{-.45x}\right]dx = -352.182776339$$

$$\int_{16.7}^{20}\left[\left(\frac{6010}{x^2-3x+50.5}\right)-8.5x^3e^{-.45x}\right]dx = 13.6653948263$$

Mary's biggest Lead 4.583

aT T=0 Mary had a 12 meTer Lead

Mary biggesT Lead

12+353.53

365.53 meTers

2018 (calc)

2. A particle moves along the x-axis with velocity given by $v(t) = \dfrac{10\sin\left(0.4t^2\right)}{t^2 - t + 3}$ for time $0 \le t \le 3.5$.

The particle is at position $x = -5$ at time $t = 0$.

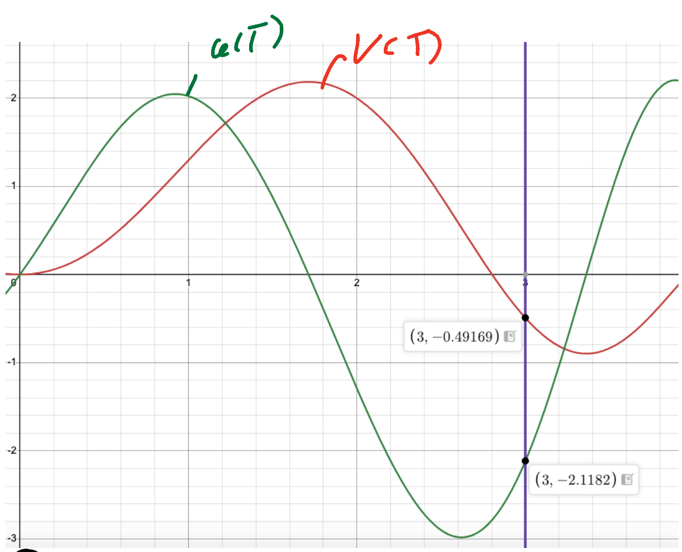
(a) Find the acceleration of the particle at time $t = 3$. $a(T) = v'(T) = -2.1182$

(b) Find the position of the particle at time $t = 3$. $S(T) = -5 + \int_0^T v(T)\,dT = -1.761$

(c) Evaluate $\int_0^{3.5} v(t)\,dt$, and evaluate $\int_0^{3.5} |v(t)|\,dt$. Interpret the meaning of each integral in the context of the problem. $\int_0^{3.5} v(T)\,dT$ = position (displacement) $\int_0^{3.5} |v(T)|\,dT$ = Total distance

(d) A second particle moves along the x-axis with position given by $x_2(t) = t^2 - t$ for $0 \le t \le 3.5$. At what time t are the two particles moving with the same velocity?

$V(T) = 2T - 1$



(b)

$$y = \int_0^3 \frac{10\sin\left(0.4x^2\right)}{x^2 - x + 3}\,dx$$

= 3.2397868128

$-5 + 3.23979$

(C)

position

$$y = \int_0^{3.5} \frac{10\sin\left(0.4x^2\right)}{x^2 - x + 3}\,dx$$

= 2.84394447491

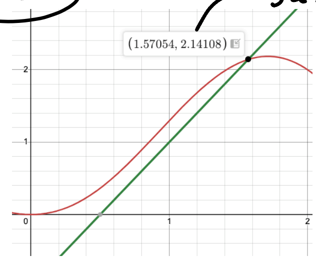
distance

$$y = \int_0^{3.5} \left|\frac{10\sin\left(0.4x^2\right)}{x^2 - x + 3}\right|dx$$

= 3.7370798155

(d)

Same velocity



2016 (calc)

2. For $t \geq 0$, a particle moves along the x-axis. The velocity of the particle at time t is given by $v(t) = 1 + 2\sin\left(\frac{t^2}{2}\right)$. The particle is at position $x = 2$ at time $t = 4$. $S(4)=2$

   (a) At time $t = 4$, is the particle speeding up or slowing down?

   (b) Find all times t in the interval $0 < t < 3$ when the particle changes direction. Justify your answer.

   (c) Find the position of the particle at time $t = 0$.

   (d) Find the total distance the particle travels from time $t = 0$ to time $t = 3$.

(a) $V(T) = 1 + 2\sin\frac{T^2}{2}$ use chain Rule

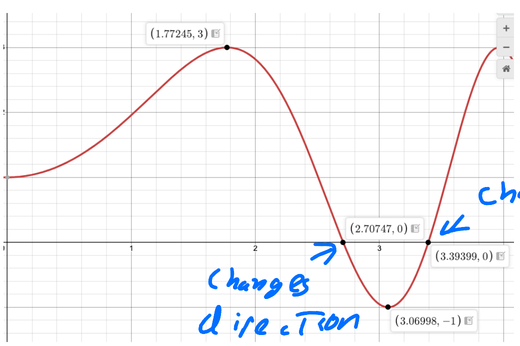
$a(T) = 0 + 2\left(\cos\frac{T^2}{2}\right)\cdot T$

$V(4) = 1 + 2\sin\frac{4^2}{2} = 2.9787$

$a(4) = 2\left(\cos\frac{4^2}{2}\right)\cdot 4 = -1.164$

different sighs
Slowing Down

(b)



changes direction

changes direction

2.707 is The only one in interval

(c) $S(4) = 2$

$\int_4^0 v(T)\,dT = y = \int_4^0 \left(1 + 2\sin\left(\frac{x^2}{2}\right)\right)dx = -5.81502683991$

$S(0) = 2 + \int_4^0 v(T)\,dT = 2 + -5.815 = -3.815$

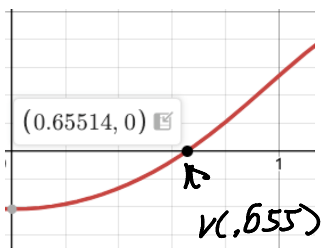
(d) $y = \int_0^3 \left|\left(1 + 2\sin\left(\frac{x^2}{2}\right)\right)\right|dx = 5.30119834688$

11. **2003 #87 (BC) - Calc OK:** A particle moves along the x-axis so that at any time $t \geq 0$, its velocity is given by $v(t) = \cos\left(2 - t^2\right)$. The position of the particle is 3 at time $t = 0$. What is the position of the particle when its velocity is first equal to 0?

$\int v(T)\,dT$

a. 0.411 b. 1.310 c. 2.816 d. 3,091 e. 3.411

$\int_0^{.65514} \cos\left(2 - x^2\right) dx = -0.183540627606$



$v(.655) = 0$

$S(0) = 3$

$S(.65514) = 3 + \int_0^{.65514} v(T)\,dT$

$3 - 0.18354 = 2.8$

12. **2003 #91 (AB but suitable for BC) - Calc OK:** A particle moves along the x-axis so that at any time $t > 0$, its acceleration is given by $a(t) = \ln\left(1 + 2^t\right)$. If the velocity of the particle is 2 at time $t = 1$, then the velocity of the particle at time $t = 2$ is

a. 0.462 b. 1.690 c. 2.555 d. 2.886 e. 3.346

$V(1) = 2$

$V(2) = 2 + \int_1^2 a(T)\,dT$

$\int_{01}^{2} \ln\left(1 + 2^x\right) dx = 1.34631353382$

$2 + 1.3463 = 3.3463$